```
eq1 = (e^(-x (1+f12)-y (1+f21)) ((-y + e^(x f12+y f21) (x + y)) f12 + y f21))/((x + y) (x f12 + y f21)) == e^(-x-y) (-1 + e^x)/((-1 + e^(x+y)) x)
```

$$\frac{e^{-x\,(1+f_{12})-y\,(1+f_{21})}\left(\left(-y+e^{x\,f_{12}+y\,f_{21}}\,(x+y)\right)\,f_{12}+y\,f_{21}\right)}{(x+y)\,(x\,f_{12}+y\,f_{21})}==\frac{e^{-x-y}\,(-1+e^{x})}{(-1+e^{x+y})\,x}$$

```
eq1 /. f21 → 0
```

$$\frac{e^{-y-x\,(1+f_{12})}\left(-y+e^{x\,f_{12}}\,(x+y)\right)}{x\,(x+y)}==\frac{e^{-x-y}\,(-1+e^{x})}{(-1+e^{x+y})\,x}$$

```
Simplify[x * (x + y) * #] & /@ (eq1 /. f21 → 0)
```

$$e^{-y-x\,(1+f_{12})}\left(-y+e^{x\,f_{12}}\,(x+y)\right)==\frac{e^{-x-y}\,(-1+e^{x})\,(x+y)}{-1+e^{x+y}}$$

```
Simplify[%]
```

$$e^{-y-x\,(1+f_{12})}\left(-y+e^{x\,f_{12}}\,(x+y)\right)==\frac{e^{-x-y}\,(-1+e^{x})\,(x+y)}{-1+e^{x+y}}$$

```
x E^(x + y) (Simplify[eq1 /. {f12 → g12 / x, f21 → g21 / y}] /. expr_ == 0 :> expr)
```

```
t1 = -(-1 + e^x)/(-1 + e^(x+y)) + (e^(-g12-g21) ((-y + e^(g12+g21) (x + y)) g12 + x g21))/((x + y) (g12 + g21))
```

$$-\frac{-1+e^{x}}{-1+e^{x+y}}+\frac{e^{-g_{12}-g_{21}}\,\left(\left(-y+e^{g_{12}+g_{21}}\,(x+y)\right)\,g_{12}+x\,g_{21}\right)}{(x+y)\,(g_{12}+g_{21})}$$

```
t1 /. g12 → 0
```

$$-\frac{-1+e^{x}}{-1+e^{x+y}}+\frac{e^{-g_{21}}\,x}{x+y}$$

```
t1 /. g21 → 0
```

$$-\frac{-1+e^{x}}{-1+e^{x+y}}+\frac{e^{-g_{12}}\,(-y+e^{g_{12}}\,(x+y))}{x+y}$$

```
Simplify[(e^(-g12-g21) ((-y + e^(g12+g21) (x + y)) g12 + x g21))/((x + y) (g12 + g21)) /. {g12 → (a + b) / 2, g21 → (a - b) / 2}]
```

$$\frac{e^{-a}\,(b\,(-1+e^{a})\,(x+y)+a\,((1+e^{a})\,x+(-1+e^{a})\,y))}{2\,a\,(x+y)}$$

```
Solve[(e^-a (b (-1 + e^a) (x + y) + a ((1 + e^a) x + (-1 + e^a) y))/(2 a (x + y)) == c, b]
```

$$\left\{\left\{b\to\frac{a\,(-x-e^{a}\,x+2\,c\,e^{a}\,x+y-e^{a}\,y+2\,c\,e^{a}\,y)}{(-1+e^{a})\,(x+y)}\right\}\right\}$$

```
Solve[(e^-a (b (-1 + e^a) (x + y) + a ((1 + e^a) x + (-1 + e^a) y))/(2 a (x + y)) == c, a]
```

Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫

$$\text{Solve}\left[\frac{e^{-a}\,(b\,(-1+e^{a})\,(x+y)+a\,((1+e^{a})\,x+(-1+e^{a})\,y))}{2\,a\,(x+y)}==c,\ a\right]$$

```
Solve[E^x == a x, x]
```

Solve::ifun : Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{x \to -\text{ProductLog}\left[-\frac{1}{a}\right]\right\}\right\}$$

```
Series[ProductLog[x], {x, 0, 10}]
```

$$x - x^2 + \frac{3 x^3}{2} - \frac{8 x^4}{3} + \frac{125 x^5}{24} - \frac{54 x^6}{5} + \frac{16\,807 x^7}{720} - \frac{16\,384 x^8}{315} + \frac{531\,441 x^9}{4480} - \frac{156\,250 x^{10}}{567} + O[x]^{11}$$

```
Rest[CoefficientList[Series[ProductLog[x], {x, 0, 10}], x]] * Range[0, 9]!
```

{1, -1, 3, -16, 125, -1296, 16 807, -262 144, 4 782 969, -100 000 000}

```
9^7
```

4 782 969

$$f_{21} = \frac{1}{4} + \left(\frac{5 x}{96} + \frac{y}{32}\right) h + \frac{\left(-99 x^3 - 187 x^2 y - 169 x y^2 - 57 y^3\right) h^3}{138\,240} +$$
$$\frac{1}{69\,672\,960}\left(780 x^5 + 2669 x^4 y + 5078 x^3 y^2 + 5000 x^2 y^3 + 2558 x y^4 + 555 y^5\right) h^5 +$$
$$\frac{1}{200\,658\,124\,800}\left(-34\,275 x^7 - 186\,139 x^6 y - 565\,943 x^5 y^2 - 949\,967 x^4 y^3 - 964\,985 x^3 y^4 - 592\,097 x^2 y^5 - 205\,357 x y^6 - 31\,989 y^7\right) h^7 +$$
$$\frac{1}{1\,589\,212\,348\,416\,000}\left(4\,104\,741 x^9 + 33\,851\,563 x^8 y + 146\,191\,048 x^7 y^2 + 352\,835\,712 x^6 y^3 + 542\,218\,746 x^5 y^4 + 552\,802\,446 x^4 y^5 + 376\,113\,312 x^3 y^6 + 165\,124\,408 x^2 y^7 + 42\,750\,073 x y^8 + 5\,104\,911 y^9\right) h^9 + O[h]^{11}$$

$$\frac{1}{4} + \left(\frac{5 x}{96} + \frac{y}{32}\right) h + \frac{\left(-99 x^3 - 187 x^2 y - 169 x y^2 - 57 y^3\right) h^3}{138\,240} + \frac{\left(780 x^5 + 2669 x^4 y + 5078 x^3 y^2 + 5000 x^2 y^3 + 2558 x y^4 + 555 y^5\right) h^5}{69\,672\,960} +$$
$$\frac{1}{200\,658\,124\,800}\left(-34\,275 x^7 - 186\,139 x^6 y - 565\,943 x^5 y^2 - 949\,967 x^4 y^3 - 964\,985 x^3 y^4 - 592\,097 x^2 y^5 - 205\,357 x y^6 - 31\,989 y^7\right) h^7 +$$
$$\frac{1}{1\,589\,212\,348\,416\,000}\left(4\,104\,741 x^9 + 33\,851\,563 x^8 y + 146\,191\,048 x^7 y^2 + 352\,835\,712 x^6 y^3 + 542\,218\,746 x^5 y^4 + 552\,802\,446 x^4 y^5 + 376\,113\,312 x^3 y^6 + 165\,124\,408 x^2 y^7 + 42\,750\,073 x y^8 + 5\,104\,911 y^9\right) h^9 + O[h]^{11}$$

```
f21 /. {x → 1, y → 1}
```

$$\frac{1}{4} + \frac{h}{12} - \frac{h^3}{270} + \frac{13 h^5}{54\,432} - \frac{431 h^7}{24\,494\,400} + \frac{27\,113 h^9}{19\,399\,564\,800} + O[h]^{11}$$

```
Log[13/54432] / 5 // N
```

-1.66795

```
Log[431/24494400] / 7 // N
```

```
-1.56398
```

```
Log[27113/19399564800] / 9 // N
```

```
-1.49786
```